

# POSTAL Book Package

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### Objective Practice Sets

#### Engineering Mathematics

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## Linear Algebra

**Q.1** Determine the rank of the following matrix :

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 0 & 0 \\ 3 & 1 & -3 & -1 \\ 1 & 1 & -3 & -1 \end{bmatrix}$$

**Q.2** The eigen vectors of the matrix  $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$  are

- (a) (4, 1) and (1, -1) (b) (0, 1) and (1, -1)  
(c) (4, 0) and (1, 0) (d) (4, 1) and (0, -1)

**Q.3** The eigen vector of the matrix  $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

corresponding to smallest eigen value is

- (a) (3, 2, 1) (b) (1, 0, 0)  
(c) (1, -1, 0) (d) None

**Q.4** By using Cayley Hamilton theorem, if  $A =$

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}, \text{ then express } A^5 - 4A^4 - 7A^3 + 11A^2 - A$$

$- 10I$  as a linear polynomial in  $A$ .

- (a)  $A + 5I$  (b)  $2A + 5I$   
(c)  $A + 7I$  (d)  $2A + 7I$

**Q.5** The determinant of the matrix  $A = \begin{bmatrix} 6 & 2 & 3 \\ 2 & 3 & 5 \\ 4 & 2 & 1 \end{bmatrix}$  is

**Q.6** The value of the determinant  $\begin{vmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{vmatrix}$

is

- (a) 0 (b) -1  
(c) 1 (d) 2

**Q.7** Eigen values of  $\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$  are

- (a) -6, -1 (b) 6, -1  
(c) -6, 1 (d) 6, 1

**Q.8** The Algebraic multiplicity of the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix} \text{ is}$$

- (a) 1 (b) 2  
(c) 3 (d) 4

**Q.9** The matrix  $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & x \end{bmatrix}$  is its own adjoint.

The value of  $x$  will be

- (a) 5 (b) 3  
(c) -3 (d) -5

**Q.10** The system of equations  $x + y + z = 6$ ,  $2x + y + z = 7$ ,  $x + 2y + z = 8$  has

- (a) A unique solution  
(b) No solution  
(c) An infinite number of solutions  
(d) None of these

**Q.11** If  $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$  is such that  $A^2 = I$ , then

- (a)  $\alpha^2 + \beta\gamma = 0$  (b)  $1 - \alpha^2 + \beta\gamma = 0$   
(c)  $1 - \alpha^2 - \beta\gamma = 0$  (d)  $1 + \alpha^2 - \beta\gamma = 0$

**Q.12** All four entries of  $2 \times 2$  matrix  $B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$

are non-zero, and one of its eigen value is zero. Which one of the following statement is true?

- (a)  $B_{11} B_{22} - B_{12} B_{21} = 1$   
(b)  $B_{11} B_{22} - B_{12} B_{21} = -1$   
(c)  $B_{11} B_{22} - B_{12} B_{21} = 0$   
(d)  $B_{11} B_{22} + B_{12} B_{21} = 0$

**Q.13** Which of the following statements are TRUE?

1. The eigen values of a Hermitian matrix are real.
  2. The value of the determinant of an orthogonal matrix can only be +1.
  3. The transpose of a square matrix  $A$  has the same eigen values as those of  $A$ .
  4. The inverse of ' $n \times n$ ' matrix exists if and only if the rank is less than ' $n$ '.
- (a) 1 and 2 only      (b) 1 and 3 only  
(c) 2 and 3 only      (d) 1 and 4 only

**Q.14** Let matrix  $[A]_{m \times n}$  has rank  $a$  and matrix  $[B]_{n \times p}$  has rank  $b$ . Then matrix  $[AB]$  may have rank

- (a)  $\leq \max(a, b)$       (b)  $n$   
(c)  $a + b$       (d)  $\leq \min(a, b)$

**Q.15** Let the eigen values of matrix  $[A]_{2 \times 2}$  are  $\alpha$  and  $\beta$  then eigen values for  $(A + 5I)^{-1}$  are

- (a)  $(\alpha + 5), (\beta + 5)$       (b)  $\frac{1}{\alpha + 5}, \frac{1}{\beta + 5}$   
(c)  $\frac{1}{\alpha} + 5, \frac{1}{\beta} + 5$       (d) can't be determined

**Q.16** Given that:  $A = \begin{bmatrix} -5 & -2 \\ 2 & 0 \end{bmatrix}$  and  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , the

value of  $A^3$  is

- (a)  $19A + 20I$       (b)  $19A + 30I$   
(c)  $21A + 20I$       (d)  $21A + 30I$

**Q.17** The determinant of the given matrix is \_\_\_\_\_.

$$M = \begin{bmatrix} 0 & 0 & 0 & 3 & 0 \\ -2 & 0 & 0 & 2 & 0 \\ 8 & -1 & 0 & -7 & 2 \\ -1 & 2 & 2 & 3 & 2 \\ 2 & 2 & 3 & 6 & 4 \end{bmatrix}$$

**Q.18** If  $A$  is an orthogonal matrix then

- (a)  $[A]^T = [A]^{-1}$       (b)  $|A| = \pm 1$   
(c)  $[A] \times [A]^T = [I]^2$       (d) All of these

**Q.19** If  $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$  then eigen values for

matrix  $A$  are

- (a)  $\pm \cos \alpha$       (b)  $\pm \sin \alpha$   
(c)  $\cos \alpha \pm \sin \alpha$       (d)  $e^{\pm i\alpha}$

**Q.20** In the matrix  $[A] = \begin{bmatrix} \sin \alpha & -\cos \alpha & 0 \\ \cos \alpha & \sin \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$  the

number of non-zero co-factors of matrix  $[A]$  is \_\_\_\_\_.

**Q.21** If  $x, y, z$  are in Arithmetic Progression (AP) with common difference ' $d$ ' and the rank of the matrix  $[A]$  is 2.

$$[A] = \begin{bmatrix} 4 & 5 & x \\ 5 & 6 & y \\ 6 & k & z \end{bmatrix}$$

The values of  $k$  and  $d$  are respectively.

- (a)  $k = 7$  and  $d$  can be any integer  
(b)  $k = \text{multiple of } 7$  and  $d$  can be any integer  
(c)  $k = \text{any multiple of } 7, d = \frac{x}{4}$   
(d)  $k = 7, d = \frac{x}{4}$

**Q.22** If the system of equations

$$\begin{aligned} ax + by + c &= 0 \\ bx + cy + a &= 0 \\ cx + ay + b &= 0 \end{aligned}$$

has a unique solution then system of equations

$$\begin{aligned} (a + b)x + (b + c)y + (c + a) &= 0 \\ (b + c)x + (c + a)y + (a + b) &= 0 \\ (c + a)x + (a + b)y + (b + c) &= 0 \end{aligned}$$

has

- (a) only one solution  
(b) no solution  
(c) infinite number of solutions  
(d) can't be determined

**Q.23** Consider the  $2 \times 2$  matrix  $\begin{bmatrix} 1 & 2 \\ p & 5 \end{bmatrix}$ . The range of

possible values of  $p$ , for which both the eigen values of the matrix are real and positive, is

- (a)  $-\frac{5}{2} \leq p \leq \frac{5}{2}$       (b)  $2 \leq p \leq \frac{5}{2}$   
(c)  $-2 \leq p \leq \frac{5}{2}$       (d)  $-\frac{5}{2} \leq p \leq 2$

**Answers Linear Algebra**

1. (2)    2. (a)    3. (c)    4. (a)    5. (-30)    6. (c)    7. (a)    8. (a)    9. (b)  
10. (a)    11. (c)    12. (c)    13. (b)    14. (d)    15. (b)    16. (c)    17. (12)    18. (d)  
19. (d)    20. (5)    21. (d)    22. (a)    23. (c)    24. (b)    25. (d)    26. (-15)    27. (b)  
28. (-22)    29. (d)    30. (9)    31. (b)    32. (b)    33. (a)    34. (6)    35. (c)

**Explanations Linear Algebra**

**1. (2)**

Given matrix

$$\sim \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 0 & 0 \\ 3 & 1 & -3 & -1 \\ 1 & 1 & -3 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (\text{Operating } R_3 - R_1, R_4 - R_1)$$

$$\sim \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{Operating } R_3 - 3R_2, R_4 -$$

$R_2$ )

(Operating  $C_3 + 3C_2, C_4 + C_2$ )

$$\sim \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = A \text{ (say)}$$

Obviously, the 4th order minor of matrix is zero.  
Also every 3rd order minor of  $A$  is zero. But, of

all the 2nd order minors, only  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = -1 \neq 0$ .

$$\therefore \rho(A) = 2$$

Hence, the rank of the given matrix is 2.

**2. (a)**

The characteristic equation is  $[A - \lambda I] = 0$

$$\text{i.e., } \begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$\text{or } \lambda^2 - 7\lambda + 6 = 0$$

$$\text{or } (\lambda - 6)(\lambda - 1) = 0$$

$$\therefore \lambda = 6, 1$$

Thus, the eigen values are 6 and 1.

If  $x, y$  be the components of an eigen vector corresponding to the eigen value  $\lambda$ , then

$$[A - \lambda I]X = \begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Corresponding to  $\lambda = 6$ , we have

$$\begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

which gives only one independent equation  
 $-x + 4y = 0$ .

$$\therefore \frac{x}{4} = \frac{y}{1} \text{ giving the eigen vector } (4, 1).$$

Corresponding to  $\lambda = 1$ , we have

$$\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

which gives only one independent equation  
 $x + y = 0$ .

$$\therefore \frac{x}{1} = \frac{y}{-1} \text{ giving the eigen vector } (1, -1).$$

**3. (c)**

The characteristic equation is

$$[A - \lambda I] = 0,$$

$$\text{i.e., } \begin{bmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{bmatrix} = 0$$

$$\text{or } (3-\lambda)(2-\lambda)(5-\lambda) = 0$$

Thus, the eigen values of  $A$  are 2, 3, 5.

If  $x, y, z$  be the components of an eigen vector corresponding to the eigen value  $\lambda$ , we have

$$[A - \lambda I]X = \begin{bmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

Putting  $\lambda = 2$ , we have  $x + y + 4z = 0$ ,  $6z = 0$ ,  
 $3z = 0$ , i.e.,  $x + y = 0$  and  $z = 0$ .

$$\therefore \frac{x}{1} = \frac{y}{-1} = \frac{z}{0} = k_1 \text{ (say)}$$

Hence, the eigen vector corresponding to  $\lambda = 2$  is  $k_1(1, -1, 0)$ .

**4. (a)**

The characteristic equation of A is

$$\begin{vmatrix} 1-\lambda & 1 \\ 2 & 3-\lambda \end{vmatrix} = 0 \text{ or } \lambda^2 - 4\lambda - 5 = 0 \quad \dots(i)$$

Now dividing the polynomial  $\lambda^5 - 4\lambda^4 - 7\lambda^3 + 11\lambda^2 - \lambda - 10I$  by the polynomial  $\lambda^2 - 4\lambda - 5$ , we obtain

$$\begin{aligned} \lambda^5 - 4\lambda^4 - 7\lambda^3 - \lambda - 10I \\ = (\lambda^2 - 4\lambda - 5)(\lambda^3 - 2\lambda + 3) + \lambda + 5 \\ = \lambda + 5 \quad [\text{By (i)}] \end{aligned}$$

Hence,  $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I = A + 5$ , which is a linear polynomial in A.

**5. (-30)**

$$\begin{aligned} \begin{vmatrix} 6 & 2 & 3 \\ 2 & 3 & 5 \\ 4 & 2 & 1 \end{vmatrix} &= 6(\text{cofactor of } 6) + 2(\text{cofactor of } 2) + 3(\text{cofactor of } 3) \\ &= 6(3 \times 1 - 5 \times 2) - 2(2 \times 1 - 4 \times 5) + 3(2 \times 2 - 3 \times 4) \\ &= 6(3 - 10) - 2(2 - 20) + 3(4 - 12) \\ &= 6(-7) - 2(-18) + 3(-8) \\ &= -42 + 36 - 24 \\ &= -30 \end{aligned}$$

**6. (c)**

$$\begin{aligned} \begin{vmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{vmatrix} \\ = \cos \theta (\cos \theta - 0) + 0 + \sin \theta (0 - (-\sin \theta)) \\ = \cos^2 \theta + \sin^2 \theta = 1 \end{aligned}$$

**7. (a)**

Characteristic equation

$$\begin{aligned} A - \lambda I &= 0 \\ \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} &= 0 \\ \begin{bmatrix} -5-\lambda & 2 \\ 2 & -2-\lambda \end{bmatrix} &= 0 \end{aligned}$$

$$(5 + \lambda)(2 + \lambda) - 4 = 0$$

$$\lambda^2 + 7\lambda + 6 = 0$$

$$\lambda^2 + 6\lambda + \lambda + 6 = 0$$

$$\lambda(\lambda + 6) + 1(\lambda + 6) = 0$$

$$\therefore \lambda = -1, -6$$

**8. (a)**

$$A = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{vmatrix} = 0 - 1(0 - 1) + 0 = 1$$

**9. (b)**

$$A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & x \end{bmatrix}$$

$$\begin{aligned} A_{11} &= \text{co-factor of } -4 = (-1)^{1+1} \begin{vmatrix} 0 & 1 \\ 4 & x \end{vmatrix} \\ &= (0 - 4) = -4 \end{aligned}$$

$$\begin{aligned} A_{12} &= \text{co-factor of } -3 = (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ 4 & x \end{vmatrix} \\ &= -(x - 4) = -x + 4 \end{aligned}$$

$$\begin{aligned} A_{13} &= \text{co-factor of } -3 = (-1)^{1+3} \begin{vmatrix} 1 & 0 \\ 4 & 4 \end{vmatrix} \\ &= 4 - 0 = 4 \end{aligned}$$

$$\begin{aligned} A_{21} &= \text{co-factor of } 1 = (-1)^{2+1} \begin{vmatrix} -3 & -3 \\ 4 & x \end{vmatrix} \\ &= -(-3x + 12) = 3x - 12 \end{aligned}$$

$$\begin{aligned} A_{22} &= \text{co-factor of } 0 = (-1)^{2+2} \begin{vmatrix} -4 & -3 \\ 4 & x \end{vmatrix} \\ &= -4x + 12 \end{aligned}$$

$$\begin{aligned} A_{23} &= \text{co-factor of } 1 = (-1)^{2+3} \begin{vmatrix} -4 & -3 \\ 4 & 4 \end{vmatrix} \\ &= 4 \end{aligned}$$

$$\begin{aligned} A_{31} &= \text{co-factor of } 4 = (-1)^{3+1} \begin{vmatrix} -3 & -3 \\ 0 & 1 \end{vmatrix} \\ &= -3 \end{aligned}$$

$$\begin{aligned} A_{32} &= \text{co-factor of } 4 = (-1)^{3+2} \begin{vmatrix} -4 & -3 \\ 1 & 1 \end{vmatrix} \\ &= 1 \end{aligned}$$

$$\begin{aligned} A_{33} &= \text{co-factor of } x = (-1)^{3+3} \begin{vmatrix} -4 & -3 \\ 1 & 0 \end{vmatrix} \\ &= 3 \end{aligned}$$